Measurement Error in Causal Inference: A Review

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Outline

Roadmap

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3 Measurement Error in Causal Inference

4 Discussion

Motivation

A necessary, but often unstated, assumption in causal inference is that all variables are measured without error

- Commonly violated; e.g. air pollution, self-reported health measures, gene expression levels

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Measurement error literature long-established...

- But work at the intersection of M.E. + causal inference is relatively new
- Growing set of methods; rationale behind them + their relative merits often unclear

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- These methods have heavily influenced early work at intersection of M.E. + causal inference

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Overview a few workhorse methods for addressing M.E. in parametric models commonly used in epi research

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Review recent developments in the causal inference literature for addressing M.E.

- Current gaps, connections to the missing data literature, and ways forward

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Attenuation Bias

Measurement error in parametric models is well-studied

- Simplest example of M.E. induces attenuation bias

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$$Y = \beta_0 + \beta_1 X + \varepsilon, \quad \varepsilon \sim N(0, \sigma^2_\varepsilon)$$
$$W = X + U, \quad \underbrace{U \sim N(0, \sigma^2_U), \quad X \perp\!\!\!\perp U}_{\text{Meas. error}}$$

Researcher observes Y, and error-prone measurements of X: W

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Researcher observes Y, and error-prone measurements of X: W

- What happens here if we ignore measurement error?

Attenuation Bias

Classical measurement error: more to the story

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Researcher observes Y, Z and W

Classical measurement error: more to the story

More General Structure

In general, the problems caused by measurement error are much more complicated than the previous pictures imply

- Error structure can be complex and systematic:

$$W \quad 0 \quad 1 \; X \quad Z \quad U$$

® ®

Meas. True Error

- Objects of interest often extend beyond the parameters of a linear regression model, e.g.

    Parameters of non-linear models
    Distribution estimation
    Causal quantities

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- Objects of interest often extend beyond the parameters of a linear regression model, e.g.
  - Parameters of non-linear models
  - Distribution estimation
  - Causal quantities

In response to these challenges, there's been a lot of work done on 1 and 2

- Work on 3 is more recent, heavily borrowing from work in 1

Contents

Identification

Return to our earlier scenario: we have data on an outcome Y, error-prone measurements W of continuous covariate X:

$$W = X + U; \quad U \sim N(0, \sigma_U^2) \text{ and } X \sim N(\mu_x, \sigma_X^2)$$
$$Y = \beta_0 + \beta_1 X + \varepsilon; \quad \varepsilon \sim N(0, \sigma_\varepsilon^2)$$

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$$Y = \beta_0 + \beta_1 X + \varepsilon; \quad \varepsilon \sim N(0, \sigma^2_\varepsilon)$$

Joint distribution of (Y, W) characterized by 5 moment equations with 6 additional unknowns on RHS (Wang 2021):

$$\mu_Y = \beta_0 + \beta_1 \mu_X$$
$$\mu_X = \mu_W$$
$$\sigma^2_Y = \beta_1 \mathrm{Cov}(Y, W) + \sigma^2_\varepsilon$$
$$\mathrm{Cov}(Y, W) = \beta_1 \sigma^2_X$$
$$\sigma^2_W = \sigma^2_X + \sigma^2_U$$

Identification + Study Design

Intuition : In order to adjust for measurement error, we need some information on the measurement error process

A few different ways to collect information on the M.E. process:

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3. Assume values/place priors on some of the M.E. params, ideally us information from previous studies

The resulting methods available for addressing M.E. are highly depende on whether (1), (2) or (3) is used

Data Structure: No Adjustments

Y	X	W	Z
Y_1		W_1	Z_1
Y_2		W_2	Z_2
Y_3		W_3	Z_3
Y_4		W_4	Z_4
Y_5		W_5	Z_5
Y_6		W_6	Z_6
Y_7		W_7	Z_7
Y_8		W_8	Z_8
Y_9		W_9	Z_9
Y_{10}		W_{10}	Z_{10}

Data Structure: Ideal Scenario

Y	X	W	Z
Y_1	X_1	W_1	Z_1
Y_2	X_2	W_2	Z_2
Y_3	X_3	W_3	Z_3
Y_4	X_4	W_4	Z_4
Y_5	X_5	W_5	Z_5
Y_6	X_6	W_6	Z_6
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Y_8	X_8	W_8	Z_8
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Data Structure: Internal Validation Data

Y	X	W	Z
Y_1	X_1	W_1	Z_1
Y_2		W_2	Z_2
Y_3	X_3	W_3	Z_3
Y_4		W_4	Z_4
Y_5	X_5	W_5	Z_5
Y_6		W_6	Z_6
Y_7	X_7	W_7	Z_7
Y_8		W_8	Z_8
Y_9	X_9	W_9	Z_9
Y_{10}		W_{10}	Z_{10}

Data Structure: Repeated Measurements

Y	X	W_1	W_2	Z
Y_1		$W_{1;1}$	$W_{2;1}$	Z_1
Y_2		$W_{1;2}$	$W_{2;2}$	Z_2
Y_3		$W_{1;3}$	$W_{2;3}$	Z_3
Y_4		$W_{1;4}$	$W_{2;4}$	Z_4
Y_5		$W_{1;5}$	$W_{2;5}$	Z_5
Y_6		$W_{1;6}$	$W_{2;6}$	Z_6
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Internal Validation Data

Mainly focus on methods that make use of internal validation data. Main reasons:

- Most M.E. adjustment methods are compatible with validation data (but many strictly require it)
- Allows for non-parametric identification of causal quantities like the average treatment effect (ATE)
  - Generally not possible without validation data
- Allows for us to use tools from the missing data literature
  - With internal validation data, M.E. becomes a missing data problem

Ways to obtain validation data: Double sampling

Other ways: Cleverness (Braun et al. 2017)

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Addressing M.E. in Parametric Models

To x ideas, suppose we'd like to estimate the parameters of the followin
model:

$$g\hat{}E\hat{}Y SX; Z \bullet\bullet \quad {}_0 \quad {}_X X \quad Z \; {}_Z$$

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$$g(E(Y|X;Z)) \quad {}_0 \quad {}_X X \quad Z \quad {}_Z$$

where we observe

$$(Y_i; W_i; Z_i); \quad i = \{1; \dots; N)$$

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To fix ideas, suppose we'd like to estimate the parameters of the following model:

$$g(E(Y|X,Z)) = \beta_0 + \beta_X X + \beta_Z Z$$

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and for a subset of subjects (say the first n) we observe

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$$W = X + U, \quad U \perp X$$

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$$W = \alpha_0 + \alpha_X X + \alpha_Z Z + U, \; (X, Z) \perp\!\!\!\perp U$$

Regression Calibration

Natural place to start: use information in the validation data to impute missing values in the main data

One way to "impute" is to just replace W with $\hat{E}(X \mid W, Z)$

- where $\hat{E}(X \mid W, Z)$ is estimated in the validation data

Regression Calibration: the main idea

Notice

$$
\begin{aligned}
E(Y|W,Z) &= E_{X|W,Z}\left[E(Y|W,X,Z)\right] \\
&= E_{X|W,Z}\left[E(Y|X,Z)\right] \qquad (Y \perp\!\!\!\perp W|X,Z) \\
&= E_{X|W,Z}\left[\beta_0 + \beta_X X + \beta_Z' Z\right] \\
&= \beta_0 + \beta_X E(X|W,Z) + \beta_Z' Z
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Under a linear outcome model, if we 1) regress X on W and Z in the validation data, and 2) replace W with $\hat{E}(X|W,Z)$ in our outcome regression, then

- we're estimating the same reg. parameters we'd estimate if we had complete information on X

Consistency hinges upon linearity of the outcome model + meas. errors not depending on Y (given X and Z)

Multiple Imputation for Measurement Error (MIME)

Likelihood Approach

Hinges on 3 assumptions

- The density of Y given Z, X is from an exponential family with dispersion parameter

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- The measurement error is additive/normally distributed:
  - $W = X + U$; $X \perp\!\!\!\perp U$; $U \sim N(0, \sigma_U^2)$

Likelihood Approach

Hinges on 3 assumptions

- The density of Y given $Z; X$ is from an exponential family with dispersion parameter
- The M.E. variance is known (!!) to be a xed value $^{2}_{U}$
- The measurement error is additive/normally distributed:
  - $W \quad X \quad U; \quad X \,⫫\, U; \quad U \quad N\,\hat{}\,0; \; {}^{2}_{U}\,\bullet$

Then, it turns out the variable

$$W \quad Y \; {}^{2}_{U} \; {}_{X} \sim$$

is su cient for X

- Can condition on Z and , solve set of score equations
- Consistent if above assumptions hold

Likelihood Approach

I.e. instead of solving score equations for

$$\mathrm{M}_{i \ 1}^{n} \ f \ \hat{} \ y_i S z_i ; x_i ; \ \bullet$$

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Solve the score equations for

$$\mathrm{M}_{i\ 1}^{n}\ f\ \hat{}\ y_i S z_i ; x_i ; \ _i ; \ \bullet \qquad \mathrm{M}_{i\ 1}^{n}\ f\ \hat{}\ y_i S z_i ; \ _i ; \ \bullet$$

L $_i$ only depends on observed data/parameters to be estimated

SIMEX

Roadmap

Contents

Background

Work on M.E. adjustment for parametric models (1980s) long pre-dates M.E. adjustment in causal inference (2010s)

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One problem...

- Growing consensus in causal inference to avoid parametric assumptions wherever possible
- By necessity, many M.E. methods need to make parametric assumptions
  - Essential when no validation data available
  - Development/application of modern semi-parametric methods has be slow

Goal: Discuss current approaches to addressing M.E. in causal researc

Background: Main Approaches

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  - A lot of methods development. Can accommodate no val. data, but requires strict parametric/distributional assumptions
- Multiple Imputation
  - Can exibly model M.E. process, easy to implement with packages lik mice and AIPW
- SIMEX (basically jackknife for measurement error)
  - Good properties when M.E. magnitude is small, but performs poorly f large magnitudes and...
  - Doesn't handle complex error structures well

Problem Setting

Three key pieces to any observational causal inference problem:

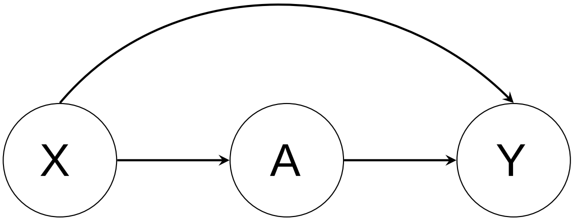
- Outcome Y
- Treatment A
- Confounders X

Measurement error can occur in any/all of them

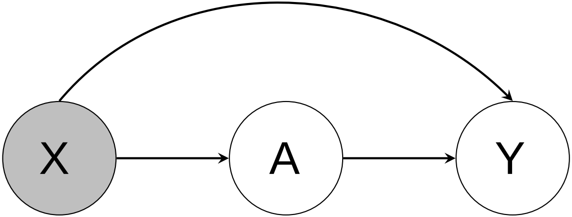
Will focus on scenario where error occurs in an important confounder

- Specifically, we'll continue to suppose we have a vector of correctly-measured confounders Z and one mis-measured confounder X (with measurements W)

Confounder Measurement Error



Confounder Measurement Error



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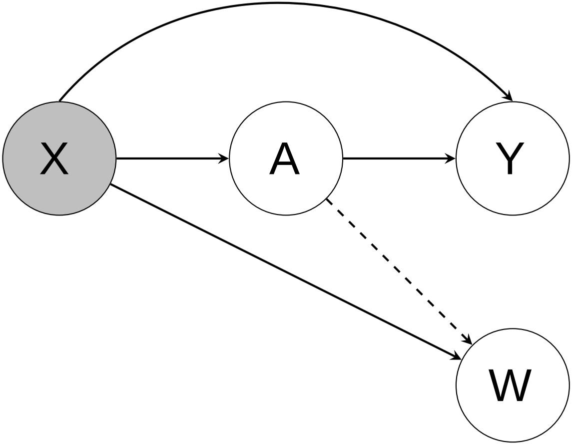

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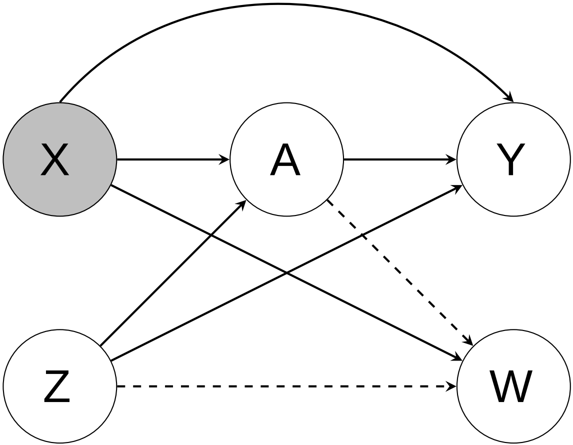

Problem Setting

To fix ideas, suppose we observe

$$(Y_i; A_i; Z_i; W_i; S_i) \sim P; \quad i = 1; \dots; N$$

and for a subset of subjects we observe

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We'd like to estimate the ATE:

$$\overset{\text{def}}{=} E(Y(1) - Y(0))$$

where $Y_i(a)$ is unit i's potential outcome had they been given treatment level a

Assumptions

Will make the following standard causal inference assumptions:

- Consistency: $Y = AY(1) + (1 - A)Y(0)$
- Unconfoundedness: $Y(a) \perp\!\!\!\perp A \mid X, Z$ (implied by DAG)
- Positivity : $P(z, x) > 0 \Rightarrow 0 < P(A = 1 \mid z, x) < 1$

Note : Unconfoundedness will not hold in observed data due to confound
M.E.

- $Y(a) \not\perp\!\!\!\perp A \mid W, Z$
- But it will hold in the validation data

Causal Identification

Under 1) consistency , 2) unconfoundedness and 3) positivity, the ATE c be identified in the validation data via

$$
\begin{aligned}
E(Y(a)\mid S=1) &= E_{X,Z\mid S=1}E(Y(a)\mid X,Z,S=1) \\
&= E_{X,Z\mid S=1}E(Y(a)\mid X,Z,A=a,S=1) \\
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&= E_{X,Z|S=1}E(Y|X,Z,A=a,S=1)
\end{aligned}
$$

If our validation data is a random sample of the main data, then

$$
E(Y(a)|S=1) = E(Y(a)) = E_{X,Z}E(Y|X,Z,A=a,S=1)
$$

- There are more useful/general identifying expressions than this (see e.g. Levis 2022)
- Without access to val. data, non-parametric identification generally not possible

Contents

Multiple Imputation

Multiple Imputation

One possible implementation:

- Estimate imputation model with exible approach, e.g. predictive mean matching
- Estimate the treatment e ect via augmented inverse probability weighting
  - Estimate the outcome and propensity score models non-parametrically o
  - Good statistical rates ($\overline{n}$ consistency) despite estimating nuisance functions with exible ML models

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Approach is consistent if imputation, complete-data ATE estimators are consistent (Nguyen and Stuart 2023)

Likelihood-based Methods

Same idea as earlier:

- Assume outcome, treatment models come from exponential families and simple M.E. structure with known (!!) variance ^2_U
- Using ^2_U and W, can construct a variable that is su cient for the unknown X
- Condition on Z and , solve score equations

Lots of papers taking version of this approach[1]

- Val. data not an option/infeasible for many applied examples
- Methods development/sensitivity analysis along these lines still important

[1] E.g. see Shu and Yi (2019); Blette (2021); McCa rey et al. (2013)

Connection to Work on Missing Data

With validation data, M.E. is really just a missing data problem

- Implying we can use tools developed for missing data problems in causal inference
- Multiple imputation is one example
- But can also take advantage of estimators developed with semi-parametric e ciency in mind
  - I.e. estimators based on e cient in uence functions

Connection to Work on Missing Data

A few examples:

- M.E. in the exposure: Kennedy (2020)
- M.E. in outcomes: Kallus and Mao (2020)
- M.E. in confounders: Levis (2022)

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- M.E. in outcome in high dimensions: Sankaranarayanan (soon)

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Basic idea: under partial missingness + causal assumptions, 1) nd identifying expression for ATE, 2) derive e . in uence function (EIF), 3) propose estimator based on EIF

- These estimators have nice properties/theoretical guarantees
  - Good statistical rates, even when nuisance models estimated with exible ML methods that themselves have slower rates
- But implementation can be quite involved

Roadmap

Discussion

- Work in causal inference for addressing M.E. still in relatively early stages
- Study design is crucial
  - To avoid heavy reliance on parametric restrictions, strive for validation data designs
- With val. data, can frame M.E. as a missing data problem
  - Can use existing tools from missing data literature, but need to keep colloborative aspect of M.E. work in mind
  - i.e. multiple imputation + AIPW easier to implement; simulation studies needed to compare with existing DR estimators
- Double sampling not always possible; continued work only assuming repeated measurements/known M.E. variance needed
  - In particular, methods for sensitivity analysis under di erent M.E. mechanisms

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