Efficient Estimation of Causal Effects Under Two-Phase Sampling with Error-Prone Outcome and Treatment Measurements

ENAR 2025 Spring Meeting

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Roadmap

Background

Methods

Data Application

Discussion

Motivation

Explosion of the use of electronic health record (EHR) data for conducting observational causal inference studies

- Where, at a high level, one is interested in some measure of causal effect of a treatment A on an outcome of interest Y

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But EHR data tends to present numerous challenges

- Including measurement error in outcomes (Y) and treatments (A) of interest

EHR data: in a perfect world

Suppose we aim to estimate the average treatment effect of a binary treatment A on Y

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Y_1	A_1	$\boldsymbol{X}_1$
Y_2	A_2	$\boldsymbol{X}_2$
Y_3	A_3	$\boldsymbol{X}_3$
Y_4	A_4	$\boldsymbol{X}_4$
Y_5	A_5	$\boldsymbol{X}_5$
Y_6	A_6	$\boldsymbol{X}_6$

In a perfect world, we'd have access to the true outcome + treatment values (+ covariates $\boldsymbol{X}$)

EHR data: in ~~a perfect world~~ reality

Y	Y^*	A	A^*	$\boldsymbol{X}$
NA	Y_1^*	NA	A_1^*	$\boldsymbol{X}_1$
NA	Y_2^*	NA	A_2^*	$\boldsymbol{X}_2$
NA	Y_3^*	NA	A_3^*	$\boldsymbol{X}_3$
NA	Y_4^*	NA	A_4^*	$\boldsymbol{X}_4$
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NA	Y_4^*	NA	A_4^*	$\boldsymbol{X}_4$
NA	Y_5^*	NA	A_5^*	$\boldsymbol{X}_5$
NA	Y_6^*	NA	A_6^*	$\boldsymbol{X}_6$

In practice, we often only have error-prone measurements of Y and A, denoted Y^* and A^*

- Well-documented that using Y^* and A^* in place of Y and A can lead to severely biased causal effect estimates

EHR data: a study design-based workaround

Y	Y^*	A	A^*	X	R
Y_1	Y_1^*	A_1	A_1^*	X_1	1
	Y_2^*		A_2^*	X_2	0
	Y_3^*		A_3^*	X_3	0
Y_4	Y_4^*	A_4	A_4^*	X_4	1
	Y_5^*		A_5^*	X_5	0
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In practice, can sometimes spend time + money to obtain gold-standard measurements for a random (typically small) subset of this data

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In practice, selection into this subset can often be controlled, and may depend on the initial error-prone measurements: $R \not\perp\!\!\!\perp A^*, Y^*, \boldsymbol{X}$

- Especially common sampling strategy if Y and/or A are rare

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- **Intuition**: over-sample subjects who contribute more information to the target estimand

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1. We have error-prone outcome + treatment measurements for all subjects
2. We have gold-standard treatment + outcome measurements for a *subset* of our EHR data, where...
3. This subset was collected according to a sampling rule that is **dependent** on the initially observed data: X, A^* and Y^* all influence R

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At a high-level, our work addresses the following question: How do we estimate causal effects nonparametrically when

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3. This subset was collected according to a sampling rule that is **dependent** on the initially observed data: X, A^* and Y^* all influence R

We present two asymptotically equivalent approaches to constructing efficient nonparametric causal effect estimators

Roadmap

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Approach 1: Using the observed data distribution

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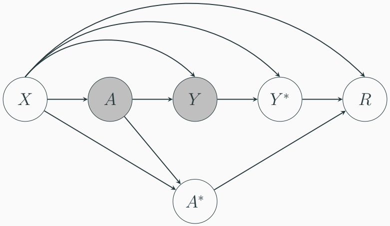

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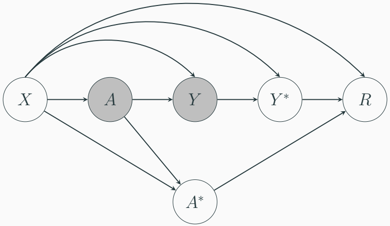

Main goal: Estimate counterfactual means $\mathbb{E}[Y(a)]$ $(a \in \{0,1\})$ *efficiently*, allowing R to depend on $\boldsymbol{X}, Y^*$ and A^*

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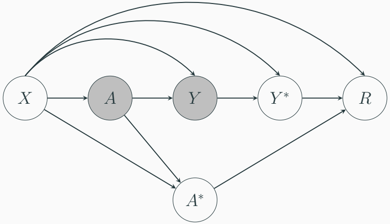

Construction of estimators follows standard semiparametric estimation pipeline:

causal estimand $\mathbb{E}[Y(a)]$ $\underset{\text{Assumptions}}{\longrightarrow}$ stat. estimand $\mathbb{E}[f(\text{data})]$ $\longrightarrow$ plug-in estimator $\hat{\psi}_a^{\text{PI}} = \frac{1}{n}\sum_{i=1}^{n} \hat{f}(\text{data}_i)$ $\underset{\text{(tedious) work}}{\longrightarrow\longrightarrow\longrightarrow}$ debiased est. $\hat{\psi}_a^{\text{OS}}$

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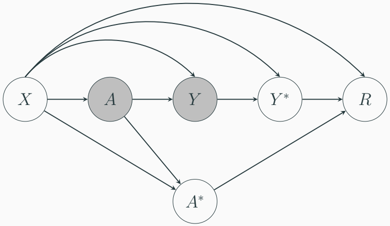

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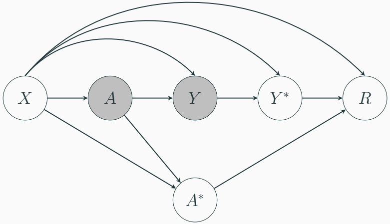

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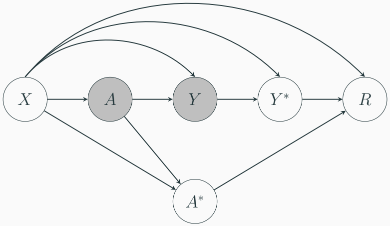

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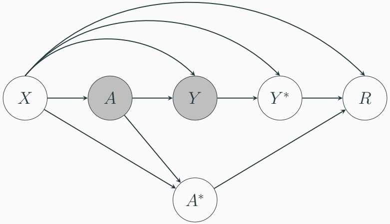

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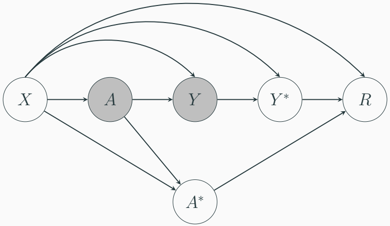

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$$\hat{\psi}_a^{\mathsf{PI,1}} = \frac{1}{n}\sum_{i=1}^{n} \frac{\hat{\mathbb{E}}[\hat{\lambda}_a(\boldsymbol{Z}) \cdot \hat{\mu}_a(\boldsymbol{Z})|\boldsymbol{X}]}{\hat{\mathbb{E}}[\hat{\lambda}_a(\boldsymbol{Z})|\boldsymbol{X}]}$$

where $\hat{\lambda}_a$ and $\hat{\mu}_a$ are imputation functions for A and Y, respectively

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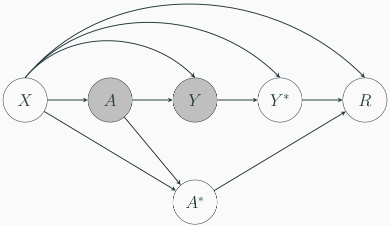

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where $\hat{\lambda}_a$ and $\hat{\mu}_a$ are imputation functions for A and Y, respectively

- Interpretation: IPW on imputed values, after marginalizing out post-treatment variables

Approach 1: Using the observed data distribution

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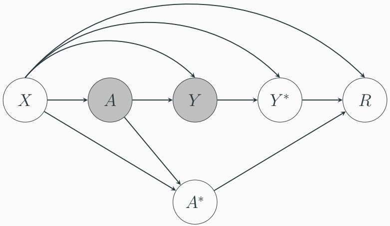

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where $\hat{\lambda}_a$ and $\hat{\mu}_a$ are imputation functions for A and Y, respectively

- Drawback: Inference intractable when nuisance models are fit data-adaptively

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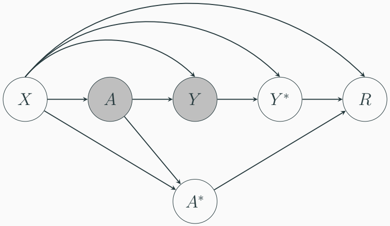

To enable inference, we derive a one-step debiased estimator

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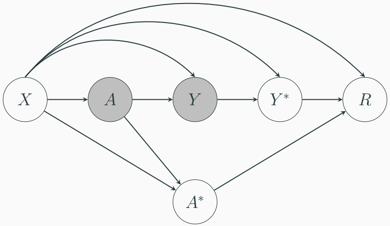

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where $\widehat{\mathsf{BC}}$ is a *bias correction* term based on the efficient influence function for ψ_a

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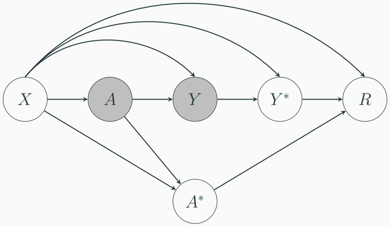

We document multiple properties of $\hat{\psi}_a^{\mathsf{OS},1}$

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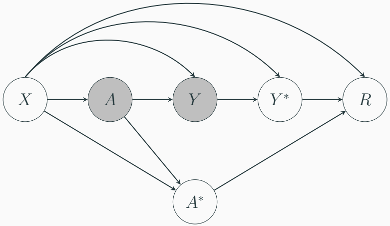

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1. Bias correction enables valid inference when nuisance models are fit with flexible ML methods that converge at $n^{1/4}$ rates, however...

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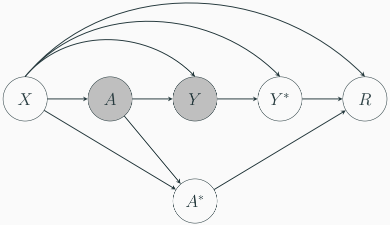

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1. Bias correction enables valid inference when nuisance models are fit with flexible ML methods that converge at $n^{1/4}$ rates, however...

2. This bias correction term introduces numerous unstable weighting terms that can harm finite sample performance
   - Particularly concerning, as validation samples tend to be small in practice

Roadmap

Approach 2: Complete-data projection

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Y_1	Y_1^*	A_1	A_1^*	X_1	1
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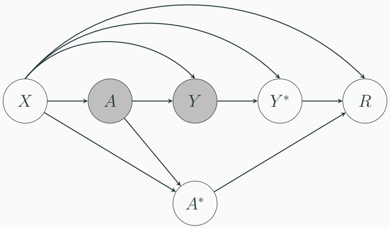

Approach 2 is based on a well-developed, but relatively underutilized, framework for constructing debiased estimators under missing data (van der Laan and Robins 2003)

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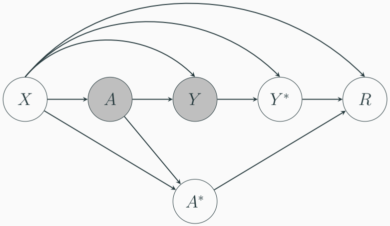

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- Links (i) the estimator we'd ideally construct under complete data to (ii) the observed data structure, where Y and A are partially missing

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Y_4	Y_4^*	A_4	A_4^*	X_4	1
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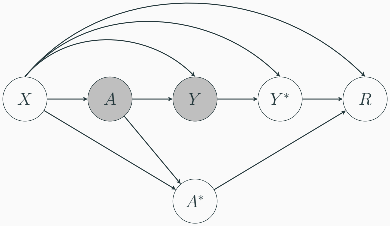
Y_6	Y_6^*	A_6	A_6^*	X_6	1



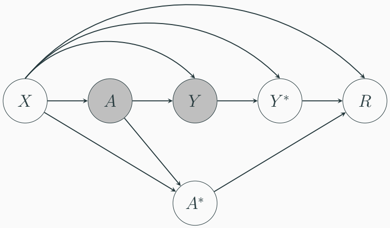
Idea: If we had complete data, could construct a plug-in estimator $\hat{\psi}_a^{\text{PI},2} = \frac{1}{n}\sum_{i=1}^{n} \hat{m}_a(\boldsymbol{X}_i)$,

$$\hat{\psi}_a^{\text{OS},1} = \hat{\psi}_a^{\text{PI},2} + \frac{1}{n}\sum_{i=1}^{n}\left(\hat{m}_a(\boldsymbol{X}_i) + \frac{I(A_i = a)}{g_a(\boldsymbol{X}_i)}\{Y_i - \hat{m}_a(\boldsymbol{X}_i)\} - \hat{\psi}_a^{\text{PI},2}\right)$$

where $m_a(\boldsymbol{X}) = \mathbb{E}(Y|A = a, \boldsymbol{X})$ and $g_a(\boldsymbol{X}) = \mathbb{P}(A = a|\boldsymbol{X})$

Approach 2: Complete-data projection

Y	Y^*	A	A^*	X	R
Y_1	Y_1^*	A_1	A_1^*	X_1	1
Y_2	Y_2^*	A_2	A_2^*	X_2	0
Y_3	Y_3^*	A_3	A_3^*	X_3	0
Y_4	Y_4^*	A_4	A_4^*	X_4	1
Y_5	Y_5^*	A_5	A_5^*	X_5	0
Y_6	Y_6^*	A_6	A_6^*	X_6	1



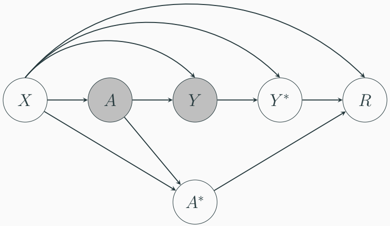
Idea: If we had complete data, could construct a plug-in estimator $\hat{\psi}_a^{\text{PI,2}} = \frac{1}{n}\sum_{i=1}^n \hat{m}_a(\boldsymbol{X}_i)$, as well as an AIPW estimator

$$\hat{\psi}_a^{\text{OS,2}} = \hat{\psi}_a^{\text{PI,2}} + \frac{1}{n}\sum_{i=1}^n \left(\hat{m}_a(\boldsymbol{X}_i) + \frac{I(A_i = a)}{\hat{g}_a(\boldsymbol{X}_i)}\{Y_i - \hat{m}_a(\boldsymbol{X}_i)\} - \hat{\psi}_a^{\text{PI,2}} \right)$$

where $m_a(\boldsymbol{X}) = \mathbb{E}(Y|A = a, \boldsymbol{X})$ and $g_a(\boldsymbol{X}) = \mathbb{P}(A = a|\boldsymbol{X})$

Approach 2: Complete-data projection

Y	Y^*	A	A^*	X	R
Y_1	Y_1^*	A_1	A_1^*	X_1	1
Y_2	Y_2^*	A_2	A_2^*	X_2	0
Y_3	Y_3^*	A_3	A_3^*	X_3	0
Y_4	Y_4^*	A_4	A_4^*	X_4	1
Y_5	Y_5^*	A_5	A_5^*	X_5	0
Y_6	Y_6^*	A_6	A_6^*	X_6	1



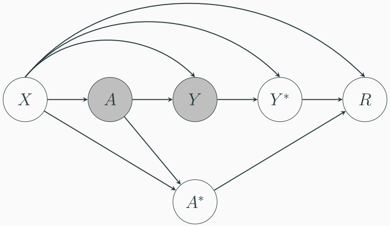
Idea: If we had complete data, could construct a plug-in estimator $\hat{\psi}_a^{\text{PI},2} = \frac{1}{n}\sum_{i=1}^n \hat{m}_a(\boldsymbol{X}_i)$, as well as an AIPW estimator

$$\hat{\psi}_a^{\text{OS},2} = \hat{\psi}_a^{\text{PI},2} + \frac{1}{n}\sum_{i=1}^n \left(\hat{m}_a(\boldsymbol{X}_i) + \frac{I(A_i = a)}{\hat{g}_a(\boldsymbol{X}_i)}\{Y_i - \hat{m}_a(\boldsymbol{X}_i)\} - \hat{\psi}_a^{\text{PI},2} \right)$$

Above, $m_a(\boldsymbol{X})$ and $\hat{g}_a(\boldsymbol{X})$ can be estimated with weighted regressions that add weights $R/\mathbb{P}(R = 1|\boldsymbol{Z})$ to the underlying loss function

Approach 2: Complete-data projection

Y	Y^*	A	A^*	X	R
Y_1	Y_1^*	A_1	A_1^*	X_1	1
Y_2	Y_2^*	A_2	A_2^*	X_2	0
Y_3	Y_3^*	A_3	A_3^*	X_3	0
Y_4	Y_4^*	A_4	A_4^*	X_4	1
Y_5	Y_5^*	A_5	A_5^*	X_5	0
Y_6	Y_6^*	A_6	A_6^*	X_6	1



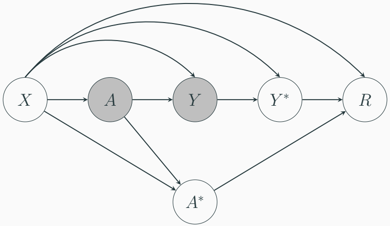
$$\hat{\psi}_a^{\mathsf{OS},2} = \hat{\psi}_a^{\mathsf{PI},2} + \frac{1}{n}\sum_{i=1}^{n}\left(\hat{m}_a(\boldsymbol{X}_i) + \frac{I(A_i = a)}{\hat{g}_a(\boldsymbol{X}_i)}\{Y_i - \hat{m}_a(\boldsymbol{X}_i)\} - \hat{\psi}_a^{\mathsf{PI},2}\right)$$

Issue: The bias correction terms are <u>only observed</u> when $R_i = 1$

- Above estimator is infeasible

Approach 2: Complete-data projection

Y	Y^*	A	A^*	X	R
Y_1	Y_1^*	A_1	A_1^*	X_1	1
Y_2	Y_2^*	A_2	A_2^*	X_2	0
Y_3	Y_3^*	A_3	A_3^*	X_3	0
Y_4	Y_4^*	A_4	A_4^*	X_4	1
Y_5	Y_5^*	A_5	A_5^*	X_5	0
Y_6	Y_6^*	A_6	A_6^*	X_6	1



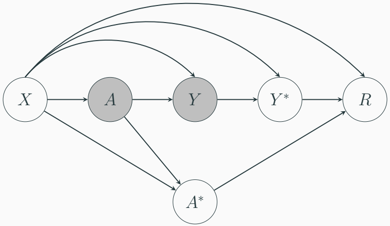
Key idea: Treat the bias correction terms as pseudo-outcomes:

$$\hat{\psi}_a^{\mathsf{OS,2}} = \hat{\psi}_a^{\mathsf{PI,2}} + \frac{1}{n}\sum_{i=1}^{n}\left(\hat{m}_a(\boldsymbol{X}_i) + \frac{I(A_i = a)}{\hat{g}_a(\boldsymbol{X}_i)}\{Y_i - \hat{m}_a(\boldsymbol{X}_i)\} - \hat{\psi}_a^{\mathsf{PI,2}}\right)$$

$$= \hat{\psi}_a^{\mathsf{PI,2}} + \frac{1}{n}\sum_{i=1}^{n}\underbrace{\chi_a(\boldsymbol{O}_i; \hat{m}_a, \hat{g}_a)}_{\text{pseudo outcome}}$$

Approach 2: Complete-data projection

Y	Y^*	A	A^*	X	R
Y_1	Y_1^*	A_1	A_1^*	X_1	1
Y_2	Y_2^*	A_2	A_2^*	X_2	0
Y_3	Y_3^*	A_3	A_3^*	X_3	0
Y_4	Y_4^*	A_4	A_4^*	X_4	1
Y_5	Y_5^*	A_5	A_5^*	X_5	0
Y_6	Y_6^*	A_6	A_6^*	X_6	1

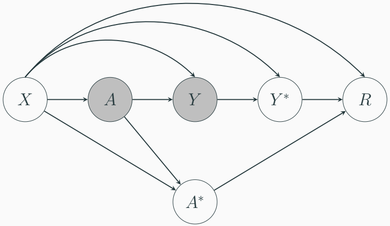


Key idea: Perform AIPW on the pseudo-outcome:

$$\hat{\psi}_a^{\mathsf{OS},2} = \hat{\psi}_a^{\mathsf{PI},2} + \frac{1}{n}\sum_{i=1}^{n} \chi_a(\boldsymbol{O}_i; \hat{m}_a, \hat{g}_a)$$

Approach 2: Complete-data projection

Y	Y^*	A	A^*	X	R
Y_1	Y_1^*	A_1	A_1^*	X_1	1
Y_2	Y_2^*	A_2	A_2^*	X_2	0
Y_3	Y_3^*	A_3	A_3^*	X_3	0
Y_4	Y_4^*	A_4	A_4^*	X_4	1
Y_5	Y_5^*	A_5	A_5^*	X_5	0
Y_6	Y_6^*	A_6	A_6^*	X_6	1



Key idea: Perform AIPW on the pseudo-outcome:

$$\hat{\psi}_a^{\mathsf{OS,2}} = \hat{\psi}_a^{\mathsf{PI,2}} + \frac{1}{n}\sum_{i=1}^{n}\left(\hat{\varphi}_a(\boldsymbol{Z}_i) + \frac{R_i}{\mathbb{P}(R_i = 1|\boldsymbol{Z}_i)}\{\chi_a(\boldsymbol{O}_i;\hat{m}_a,\hat{g}_a) - \hat{\varphi}_a(\boldsymbol{Z}_i)\}\right)$$

Approach 2: Complete-data projection

Y	Y^*	A	A^*	X	R
Y_1	Y_1^*	A_1	A_1^*	X_1	1
Y_2	Y_2^*	A_2	A_2^*	X_2	0
Y_3	Y_3^*	A_3	A_3^*	X_3	0
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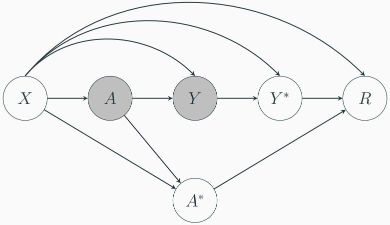

Key idea: Perform AIPW on the pseudo-outcome:

$$\hat{\psi}_a^{\text{OS},2} = \hat{\psi}_a^{\text{PI},2} + \frac{1}{n}\sum_{i=1}^{n}\left(\hat{\varphi}_a(\boldsymbol{Z}_i) + \frac{R_i}{\mathbb{P}(R_i = 1|\boldsymbol{Z}_i)}\{\chi_a(\boldsymbol{O}_i; \hat{m}_a, \hat{g}_a) - \hat{\varphi}_a(\boldsymbol{Z}_i)\}\right)$$

where $\hat{\varphi}_a(\boldsymbol{Z}) = \hat{\mathbb{E}}[\chi_a(\boldsymbol{O}; \hat{m}_a, \hat{g}_a)|\boldsymbol{Z}, R = 1]$

Approach 2: Complete-data projection

Y	Y^*	A	A^*	X	R
Y_1	Y_1^*	A_1	A_1^*	X_1	1
Y_2	Y_2^*	A_2	A_2^*	X_2	0
Y_3	Y_3^*	A_3	A_3^*	X_3	0
Y_4	Y_4^*	A_4	A_4^*	X_4	1
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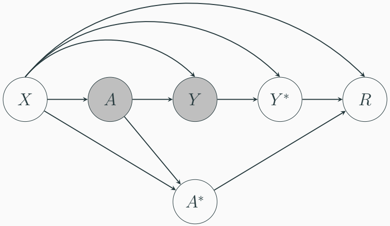

Key idea: Perform AIPW on the pseudo-outcome:

$$\hat{\psi}_a^{\mathsf{OS},2} = \hat{\psi}_a^{\mathsf{PI},2} + \frac{1}{n}\sum_{i=1}^{n}\left(\hat{\varphi}_a(\boldsymbol{Z}_i) + \frac{R_i}{\mathbb{P}(R_i = 1|\boldsymbol{Z}_i)}\{\chi_a(\boldsymbol{O}_i; \hat{m}_a, \hat{g}_a) - \hat{\varphi}_a(\boldsymbol{Z}_i)\}\right)$$

where $\hat{\varphi}_a(\boldsymbol{Z}) = \mathbb{E}[\chi_a(\boldsymbol{O}_i; \hat{m}_a, \hat{g}_a)|\boldsymbol{Z}, R = 1]$

- Feasible estimator!
- When sampling probabilities are known, $\hat{\psi}_a^{\mathsf{OS},2}$ is doubly-robust in the traditional sense

Approach 2: Complete-data projection

Y	Y^*	A	A^*	X	R
Y_1	Y_1^*	A_1	A_1^*	X_1	1
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Y_5	Y_5^*	A_5	A_5^*	X_5	0
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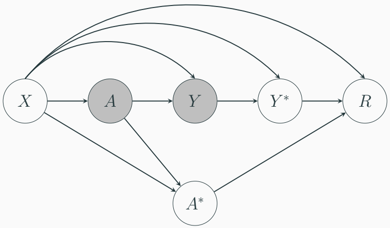

Connections between $\hat{\psi}_a^{\mathsf{OS},1}$ and $\hat{\psi}_a^{\mathsf{OS},2}$:

- Can be shown that asymptotically, Approach 1 and Approach 2 estimators are *equivalent*
  - Approach 2 can be viewed as a re-parameterization of Approach 1
- Both estimators have unique sources of finite sample instability

Roadmap

Background

Methods

Data Application

Discussion

The Vanderbilt Comprehensive Care Clinic (VCCC)

EHR database with information on $\approx$ 1900 patients living with HIV between 1998-2011 that began receiving care at the VCCC

- Numerous variables recorded with error
    - Date of antiretroviral therapy (ART) initiation
    - Occurrence of AIDS-defining events (ADEs)

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- Team at Vanderbilt University Medical Center manually validated *every* patient's observation in the database
  - Effectively yielding a fully validated dataset alongside an error-prone dataset

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- Team at Vanderbilt University Medical Center manually validated *every* patient's observation in the database
  - Effectively yielding a fully validated dataset alongside an error-prone dataset

Causal estimand: Average causal effect of starting ART within 1 month of first visit (A) on 3-year (post initial visit) risk of suffering an ADE (Y)

VCCC application

"Reveal" increasingly larger shares of the validated data $\mathbb{P}(R = 1)$, over-sampling those with $A^* = 1$ and $Y^* = 1$

- E.g. 5%, 10%, $15\%, \ldots$
- Implement proposed estimators at each share

Y	Y^*	A	A^*	X	R
Y_1	Y_1^*	A_1	A_1^*	X_1	1
	Y_2^*		A_2^*	X_2	0
	Y_3^*		A_3^*	X_3	0
	Y_4^*		A_4^*	X_4	0
	Y_5^*		A_5^*	X_5	0
	Y_6^*		A_6^*	X_6	0
	Y_7^*		A_7^*	X_7	0
	Y_8^*		A_8^*	X_8	0
	Y_9^*		A_9^*	X_9	0
	Y_{10}^*		A_{10}^*	X_{10}	0

VCCC application

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	Y_3^*		A_3^*	X_3	0
	Y_4^*		A_4^*	X_4	0
	Y_5^*		A_5^*	X_5	0
	Y_6^*		A_6^*	X_6	0
	Y_7^*		A_7^*	X_7	0
	Y_8^*		A_8^*	X_8	0
	Y_9^*		A_9^*	X_9	0
	Y_{10}^*		A_{10}^*	X_{10}	0

VCCC application

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	Y_5^*		A_5^*	X_5	0
	Y_6^*		A_6^*	X_6	0
	Y_7^*		A_7^*	X_7	0
	Y_8^*		A_8^*	X_8	0
	Y_9^*		A_9^*	X_9	0
	Y_{10}^*		A_{10}^*	X_{10}	0

VCCC application

"Reveal" increasingly larger shares of the validated data $\mathbb{P}(R = 1)$, over-sampling those with $A^* = 1$ and $Y^* = 1$

- E.g. 5%, 10%, $15\%, \ldots$
- Implement proposed estimators at each share

Y	Y^*	A	A^*	X	R
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Y_2	Y_2^*	A_2	A_2^*	X_2	1
Y_3	Y_3^*	A_3	A_3^*	X_3	1
Y_4	Y_4^*	A_4	A_4^*	X_4	1
	Y_5^*		A_5^*	X_5	0
	Y_6^*		A_6^*	X_6	0
	Y_7^*		A_7^*	X_7	0
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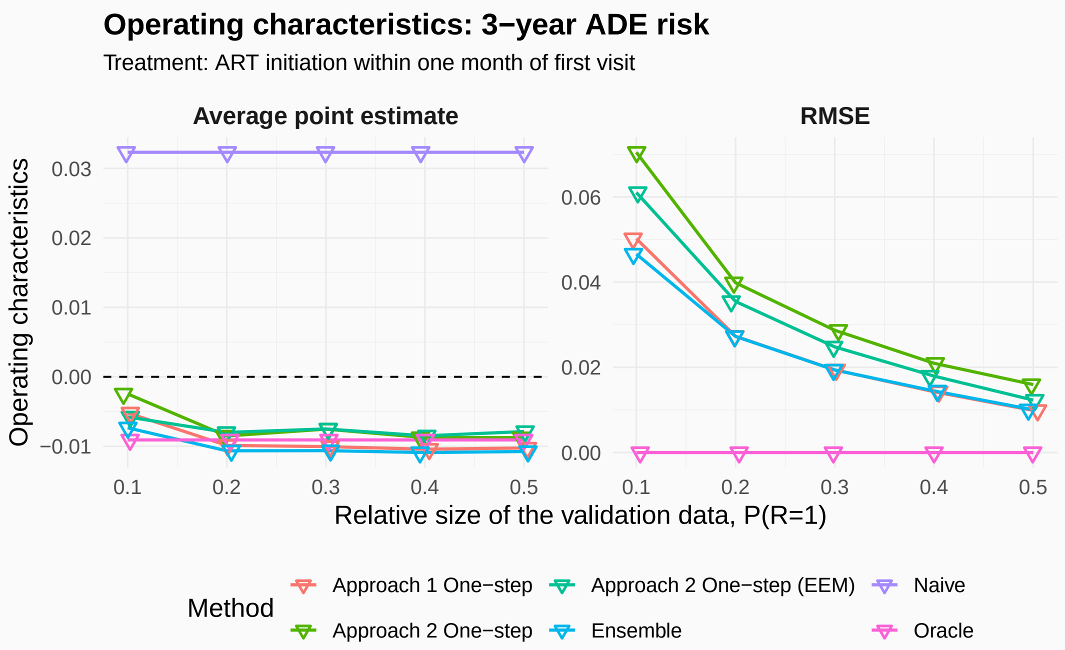
	Y_{10}^*		A_{10}^*	X_{10}	0


Operating characteristics: 3−year ADE risk
Treatment: ART initiation within one month of first visit
Average point estimate
RMSE
Operating characteristics
0.03
0.02
0.01
0.00
−0.01
0.06
0.04
0.02
0.00
0.1
0.2
0.3
0.4
0.5
Relative size of the validation data, P(R=1)
Method
Approach 1 One−step
Approach 2 One−step (EEM)
Naive
Approach 2 One−step
Ensemble
Oracle

Roadmap

Discussion

Our work addresses the following high-level question:

How do we estimate causal effects nonparametrically when **(1)** we have error-prone outcome + treatment measurements, **(2)** we have gold-standard treatment + outcome measurements for a *subset* of our data, where **(3)** that subset was collected according to a sampling rule that depends on the initially observed data

We've presented plug-in + debiased one-step estimators that accommodate these sampling schemes (and general two-phase sampling schemes). Currently working on

- Further studying approaches that improve the finite-sample behavior of the one-step estimators
- Development of R package implementing Approach 2 for general missing data patterns

Thank you!

Working paper coming soon!

References

van der Laan, M. J. and Robins, J. M. (2003). *Unified methods for censored longitudinal data and causality*. Springer.

Appendix

Identification

First, let $\boldsymbol{W} = (Y^*, A^*)$. Then, we have

$$\begin{aligned}
\mathbb{E}(Y(a)) &= \mathbb{E}_{\boldsymbol{X}}\mathbb{E}(Y(a)|\boldsymbol{X}) \\
&= \mathbb{E}_{\boldsymbol{W}(a)|\boldsymbol{X}}\left[\mathbb{E}_{\boldsymbol{X}}\mathbb{E}(Y(a)|\boldsymbol{X}, \boldsymbol{W}(a))\right] \\
&= \mathbb{E}_{\boldsymbol{W}(a)|\boldsymbol{X}}\left[\mathbb{E}_{\boldsymbol{X}}\mathbb{E}(Y(a)|\boldsymbol{X}, \boldsymbol{W}(a), A = a, R = 1)\right] \\
&= \mathbb{E}_{\boldsymbol{W}|\boldsymbol{X},A}\left[\mathbb{E}_{\boldsymbol{X}}\mathbb{E}(Y|\boldsymbol{X}, \boldsymbol{W}, A = a, R = 1)\right]
\end{aligned}$$

The problem: the density $p(w|x, a)$ is unidentified

- We only see A in the validation data, which is **not** independent of $\boldsymbol{W}$ so we can't just condition on $R = 1$

Identification

Notice

$$\mathbb{E}(Y(a)) = \sum_w \sum_x \sum_y y \cdot p(y|w, x, a, r = 1) \cdot p(w|x, a) \cdot p(x)$$

Identification

Notice

$$\begin{aligned}
\mathbb{E}(Y(a)) &= \sum_w \sum_x \sum_y y \cdot p(y|w,x,a,r=1) \cdot p(w|x,a) \cdot p(x) \\
&= \sum_w \sum_x \sum_y y \cdot p(y|w,x,a,r=1) \cdot \frac{p(a|x,w)p(w|x)p(x)}{p(a|x)p(x)} \cdot p(x)
\end{aligned}$$

Identification

Notice

$$
\begin{aligned}
\mathbb{E}(Y(a)) &= \sum_w \sum_x \sum_y y \cdot p(y|w,x,a,r=1) \cdot p(w|x,a) \cdot p(x) \\
&= \sum_w \sum_x \sum_y y \cdot p(y|w,x,a,r=1) \cdot \frac{p(a|x,w)p(w|x)p(x)}{p(a|x)p(x)} \cdot p(x) \\
&= \sum_w \sum_x \sum_y y \cdot p(y|w,x,a,r=1) \cdot \frac{p(a|x,w,r=1)p(w|x)}{\sum_{w'} p(a|w',x)p(w'|x)} \cdot p(x)
\end{aligned}
$$

Identification

Notice

$$\begin{aligned}\mathbb{E}(Y(a)) &= \sum_w \sum_x \sum_y y \cdot p(y|w,x,a,r=1) \cdot p(w|x,a) \cdot p(x) \\ &= \sum_w \sum_x \sum_y y \cdot p(y|w,x,a,r=1) \cdot \frac{p(a|x,w)p(w|x)p(x)}{p(a|x)p(x)} \cdot p(x) \\ &= \sum_w \sum_x \sum_y y \cdot p(y|w,x,a,r=1) \cdot \frac{p(a|x,w,r=1)p(w|x)}{\sum_{w'} p(a|w',x)p(w'|x)} \cdot p(x)\end{aligned}$$

Key idea: Use Bayes rule to re-express un-identified density in terms of identifiable ones

- Then, overall expression is identified – just need to define terms and "collapse" the above expectations

Plug-in implementation

Y_i	Y_i^*	A_i	A_i^*	$\boldsymbol{X}_i$	R_i	$\hat{\lambda}_a$	$\hat{\mu}_a$	$\hat{\pi}_a$	$\hat{\eta}_a$
1	0	0	1	3.2	1	NA	NA	NA	NA
0	1	1	1	2.1	1	NA	NA	NA	NA
1	1	0	0	0.3	1	NA	NA	NA	NA
1	0	0	1	1.6	0	NA	NA	NA	NA
0	1	0	1	4.8	0	NA	NA	NA	NA
1	1	0	0	0.9	0	NA	NA	NA	NA

Step 1: use validation data to fit $\hat{\lambda}_a$ and $\hat{\mu}_a$

Plug-in implementation

Y_i	Y_i^*	A_i	A_i^*	$\boldsymbol{X}_i$	R_i	$\hat{\lambda}_a$	$\hat{\mu}_a$	$\hat{\pi}_a$	$\hat{\eta}_a$
1	0	0	1	3.2	1	0.2	0.72	NA	NA
0	1	1	1	2.1	1	0.43	0.18	NA	NA
1	1	0	0	0.3	1	0.64	0.77	NA	NA
1	0	0	1	1.6	0	0.03	0.55	NA	NA
0	1	0	1	4.8	0	0.81	0.11	NA	NA
1	1	0	0	0.9	0	0.39	0.83	NA	NA

Step 1: use validation data to fit $\hat{\lambda}_a$ and $\hat{\mu}_a$

Plug-in implementation

Y_i	Y_i^*	A_i	A_i^*	$\boldsymbol{X}_i$	R_i	$\hat{\lambda}_a$	$\hat{\mu}_a$	$\hat{\pi}_a$	$\hat{\eta}_a$
1	0	0	1	3.2	1	0.2	0.72	`NA`	`NA`
0	1	1	1	2.1	1	0.43	0.18	`NA`	`NA`
1	1	0	0	0.3	1	0.64	0.77	`NA`	`NA`
1	0	0	1	1.6	0	0.03	0.55	`NA`	`NA`
0	1	0	1	4.8	0	0.81	0.11	`NA`	`NA`
1	1	0	0	0.9	0	0.39	0.83	`NA`	`NA`

Step 2: regress $\hat{\lambda}_a$ and $\hat{\mu}_a$ on $\boldsymbol{X}$ and A^* to yield $\hat{\pi}_a$ and $\hat{\eta}_a$

Plug-in implementation

Y_i	Y_i^*	A_i	A_i^*	$\boldsymbol{X}_i$	R_i	$\hat{\lambda}_a$	$\hat{\mu}_a$	$\hat{\pi}_a$	$\hat{\eta}_a$
1	0	0	1	3.2	1	0.2	0.72	0.77	0.23
0	1	1	1	2.1	1	0.43	0.18	0.41	0.26
1	1	0	0	0.3	1	0.64	0.77	0.59	0.80
1	0	0	1	1.6	0	0.03	0.55	0.08	0.53
0	1	0	1	4.8	0	0.81	0.11	0.72	0.18
1	1	0	0	0.9	0	0.39	0.83	0.38	0.79

Step 2: regress $\hat{\lambda}_a$ and $\hat{\mu}_a$ on $\boldsymbol{X}$ and A^* to yield $\hat{\pi}_a$ and $\hat{\eta}_a$

Plug-in implementation

Y_i	Y_i^*	A_i	A_i^*	$\boldsymbol{X}_i$	R_i	$\hat{\lambda}_a$	$\hat{\mu}_a$	$\hat{\pi}_a$	$\hat{\eta}_a$
1	0	0	1	3.2	1	0.2	0.72	0.77	0.23
0	1	1	1	2.1	1	0.43	0.18	0.41	0.26
1	1	0	0	0.3	1	0.64	0.77	0.59	0.80
1	0	0	1	1.6	0	0.03	0.55	0.08	0.53
0	1	0	1	4.8	0	0.81	0.11	0.72	0.18
1	1	0	0	0.9	0	0.39	0.83	0.38	0.79

Step 3: construct $\hat{\psi}_a^{\mathsf{PI}} = \frac{1}{n}\sum_{i=1}^{n} \frac{\hat{\eta}_a(\boldsymbol{X}_i, A_i^*)}{\hat{\pi}_a(\boldsymbol{X}_i, A_i^*)}$

Equivalence of Approach 1 and Approach 2 EICs

$$\begin{aligned}\phi_a^{\mathsf{obs}}(\boldsymbol{O}) &= \frac{R}{\mathbb{P}(R=1|\boldsymbol{Z})}\chi_a^{\mathsf{full}}(\boldsymbol{O}) \\ &\quad - \left(\frac{R}{\mathbb{P}(R=1|\boldsymbol{Z})} - 1\right)\varphi_a(\boldsymbol{Z})\end{aligned}$$

Equivalence of Approach 1 and Approach 2 EICs

$$\phi_a^{\mathsf{obs}}(\boldsymbol{O}) = \frac{R}{\mathbb{P}(R=1|\boldsymbol{Z})}\left[\frac{I(A=a)}{e_a(\boldsymbol{X})}(Y - m_a(\boldsymbol{X})) + m_a(\boldsymbol{X}) - \psi_a\right]$$
$$- \left(\frac{R}{\mathbb{P}(R=1|\boldsymbol{Z})} - 1\right)\varphi_a(\boldsymbol{Z})$$

Equivalence of Approach 1 and Approach 2 EICs

$$\phi_a^{\mathsf{obs}}(\boldsymbol{O}) = \frac{R}{\mathbb{P}(R=1|\boldsymbol{Z})}\left[\frac{I(A=a)}{\pi_a(\boldsymbol{X})}\left(Y - \frac{\eta_a(\boldsymbol{X})}{\pi_a(\boldsymbol{X})}\right) + \frac{\eta_a(\boldsymbol{X})}{\pi_a(\boldsymbol{X})} - \psi_a\right]$$
$$- \left(\frac{R}{\mathbb{P}(R=1|\boldsymbol{Z})} - 1\right)\varphi_a(\boldsymbol{Z})$$

Equivalence of Approach 1 and Approach 2 EICs

$$\begin{aligned}
\phi_a^{\mathsf{obs}}(\boldsymbol{O}) &= \frac{R}{\mathbb{P}(R=1|\boldsymbol{Z})}\left[\frac{I(A=a)}{\pi_a(\boldsymbol{X})}\left(Y-\frac{\eta_a(\boldsymbol{X})}{\pi_a(\boldsymbol{X})}\right)+\frac{\eta_a(\boldsymbol{X})}{\pi_a(\boldsymbol{X})}-\psi_a\right] \\
&\quad -\left(\frac{R}{\mathbb{P}(R=1|\boldsymbol{Z})}-1\right)\left(\frac{\lambda_a(\boldsymbol{Z})\mu_a(\boldsymbol{Z})}{\pi_a(\boldsymbol{X})}-\frac{\lambda_a(\boldsymbol{Z})\eta_a(\boldsymbol{X})}{\pi_a(\boldsymbol{X})^2}+\frac{\eta_a(\boldsymbol{X})}{\pi_a(\boldsymbol{X})}-\psi_a\right)
\end{aligned}$$